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Identification of modal parameters from ambient vibration data using eigensystem realization algorithm with correlation technique †

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Abstract

An effective identification method is developed for the determination of modal parameters of a structure based on the measured ambient response data. In this study, modification to Eigensystem Realization Algorithm with Data Correlation is proposed for modal-parameter identification of structural systems subjected to stationary white-noise ambient vibration. By setting up a correlation -function matrix of stationary responses, as well as by introducing an appropriate matrix factorization, modal parameters of a system can be identified effectively through singular -value decomposition and eigenvalue analysis. Numerical simulations using practical excitation data confirm the validity and robustness of the proposed method in identifying modal parameters from stationary ambient vibration data under noisy conditions.

Keywords: Eigensystem Realization Algorithm with Data Correlation (ERA/DC); Correlation technique; Stationary ambient vibration

1. Introduction

In general, identification of system characteristics is accomplished using both input and output data from a structural system. However, in many cases, only measured output data can be used for structures under ambient vibration conditions. Hence it is beneficial to develop applicable methods for modal -parameter identification directly from ambient vibration data (i.e., without the need for input measurement).

Numerous papers have been presented on system identification, acquiring the estimation of important parameters from measured data. Ho and Kalman [1] introduced an important principle in minimal realization theory in which a state-space model using the Hankel matrix is constructed by a sequence of Markov parameters (impulse response functions) from the system. However, error characteristics due to noise are required for estimation. Zeiger and McEwen [2] proposed a concept combining singular-value decomposition (SVD) and minimal realization algorithm. Based on the developments of SVD and minimal realization algorithm, the well-known Eigensystem Realization Algorithm (ERA), proposed by Juang and Pappa [3], was developed to realize the unknown system, given the noisy input-output measurements. The algorithm also provided accurate parameter estimation and system order determination for multivariable linear state-space models. Juang et al. also proposed a modification of ERA, generally known as Eigensystem Realization Algorithm with Data Correlation (ERA/DC) [4], using data correlations to reduce the noise effect in modal -parameter identification. James et al. [5] also developed the so-called Natural Excitation Technique (NExT) that uses correlation functions of measured response combined with a time-domain parameter extraction scheme under the assumption of white-noise input. Chiang and Cheng [6] presented a correlation technique for modal-parameter identification of a linear, complex -mode system subjected to stationary ambient excitation. Chiang and Lin [7] further extended the correlation technique for modal-parameter identification of a system excited by *non-stationary* white noise in the form of a product model.

This paper presents a modification to the Eigensystem Realization Algorithm with Data Correlation (MERA/DC) for linear systems excited by stationary white -noise ambient vibration. By setting up a correlation -function matrix of the stationary ambient responses and introducing an appropriate matrix factorization, modal parameters of a system can be identified using the SVD and eigenvalue analysis. Numerical simulations of using practical excitation data confirm the validity and robustness of the proposed method in the identification of modal parameters from stationary ambient vibration data under noisy conditions.

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2. Correlation technique

James et al. [5] developed the so-called NExT using the correlation technique. When a system is excited by stationary white noise, the cross-correlation function $R_{ij}(\tau)$ between two stationary response signals $x_i(t)$ and $x_j(t)$ can be shown as [6]

$$R_{ij}(\tau)=\sum_{r=1}^{n}\frac{\phi_{ir}A_{jr}}{m_r\,\omega_{dr}}\exp(-\zeta_r\,\omega_r\tau)\sin(\omega_{dr}\,\tau+\theta_r)\ . \tag{1}$$

where ϕ_{ir} is the *i*-th component of the *r*-th mode shape, A_{jr} is a constant, and m_r is the *r*-th modal mass. The above result shows that $R_{ij}(\tau)$ in Eq. (1) is a sum of complex exponential functions (modal responses), which is of the same mathematical form as the free vibration decay or the impulse response of the original system. Thus, the cross-correlation functions evaluated from response data can be used as free vibration decay or as impulse response in time-domain modal extraction schemes to avoid measurement of white -noise inputs. It is remarkable that the term $\phi_{ir}A_{jr}$ in Eq. (1) is identified as mode -shape components. In order to eliminate the A_{jr} term and retain the true mode -shape components ϕ_{ir}, all the measured channels are correlated against a common reference channel, say x_j. Then, identified components all possess the common A_{jr} component, which can be normalized to obtain the desired mode shape ϕ_{ir}.

3. Modified eigensystem realization algorithm with data correlation (MERA/DC)

Juang et al. [4] proposed a modification of ERA called ERA/DC, which uses data correlations to reduce the noise effect for modal-parameter identification. As shown in the previous section, the correlation functions between two stationary response signals of a structure subjected to white-noise inputs can be treated as free vibration decay or as impulse response for further extraction of modal parameters. Thus, the correlation functions can be used to extract modal parameters of a system in the case of stationary ambient vibration. A modification to the Eigensystem Realization Algorithm with Data Correlation is presented for linear systems excited by stationary white-noise ambient vibrations. By setting up a correlation - function matrix of the stationary responses and introducing a proper matrix factorization, modal parameters of a system can be identified through SVD and eigenvalue analysis.

For finite but sufficiently long records, say of l time steps, the correlation- function matrix $\boldsymbol{T}_\tau$ is defined and is composed of measured responses from every channel with different time-delayed signals. $\boldsymbol{T}_\tau$ can be approximately expressed as

$$\boldsymbol{T}_\tau\equiv\boldsymbol{T}(\tau)\approx\frac{1}{l}\left[\boldsymbol{Y}_{k+\tau}\boldsymbol{Y}_k^T\right]_{p\times p} \tag{2}$$

where $\boldsymbol{Y}_k=\{y_{ij}, i=1\sim p, j=k-l\sim k\}$ signifies the measured responses of a system subjected to the excitation of a zero-mean stationary white noise $\boldsymbol{u}_k=\{u_i,\, i=1\sim k\}$ at the time - step k, and p is the channel number of the measured response. The correlation-function matrix $\boldsymbol{T}_\tau$ of the measured system response can be obtained from a single sample function of time by using the ergodic property of stationary random processes. $\boldsymbol{T}_\tau$ can be viewed as impulse functions (also known as the Markov parameters) in the conventional ERA/DC process and can be expressed as [8]

$$\boldsymbol{T}_\tau\approx\boldsymbol{C}\boldsymbol{A}^{\tau-1}\boldsymbol{G}\ , \tag{3}$$

where $\boldsymbol{C}$ is the $p\times n$ output influence matrix for the state vector, $\boldsymbol{A}$ is the $n\times n$ state matrix with dynamic characteristics of the system, and $\boldsymbol{G}\approx\frac{1}{l}\left[\boldsymbol{X}_{k+1}\boldsymbol{Y}_k^T\right]_{n\times p}$ is the so-called covariance matrix, where $\boldsymbol{X}_{k+1}$ is the $n\times l$ state-vectors matrix of the system. It follows from almost the same procedure as used in ERA/DC [4] to define the data-correlation matrix $\overline{\boldsymbol{R}}(k)$, which is formed using the correlation-function matrix $\boldsymbol{T}_\tau$. A generalized Hankel matrix $\overline{\boldsymbol{U}}(k)$ using the data-correlation matrix $\overline{\boldsymbol{R}}(k)$ is to reduce the effect of noise, and to increase accuracy of the parameters estimation. Based on the system realization theory, ERA/DC constructs a discrete state-space model of minimal order using SVD analysis. To determine the system order, which is defined as the number of structural modes involved in the responses, SVD of the generalized Hankel matrix $\overline{\boldsymbol{U}}(k)$ is performed. The rank of the generalized Hankel matrix $\overline{\boldsymbol{U}}(k)$ is the system order, which is just the number of the obvious non-zero n singular values. Therefore, the state matrix $\boldsymbol{A}$ can be obtained by retaining the first n sub-vectors as follows:

$$\boldsymbol{A}=\overline{\boldsymbol{S}}_n^{-1/2}\overline{\boldsymbol{V}}_n^{T}\,\overline{\boldsymbol{U}}(1)\,\overline{\boldsymbol{W}}_n\overline{\boldsymbol{S}}_n^{-1/2}\ , \tag{4}$$

where $\overline{\boldsymbol{V}}_n$ and $\overline{\boldsymbol{W}}_n$ are the matrices with orthonormal property, i.e., $\overline{\boldsymbol{V}}_n^T\overline{\boldsymbol{V}}_n=\boldsymbol{I}_n=\overline{\boldsymbol{W}}_n^T\overline{\boldsymbol{W}}_n$, and $\overline{\boldsymbol{S}}_n$ is a diagonal matrix containing the n largest singular values. The matrices of eigenvalues and eigenvectors of the *n*–order state matrix $\boldsymbol{A}$ are denoted as $\boldsymbol{Z}=diag(z_1,\, z_2,\, \cdots,\, z_n)$ and $\boldsymbol{\Psi}=[\psi_1,\, \psi_2,\, \cdots,\, \psi_n]$, respectively. The eigenvalues contain the information of modal frequencies and damping ratios. Thus, when the state matrix $\boldsymbol{A}$ is obtained from measured data, the modal parameters of the structure system of interest can be determined by solving the eigenvalue problem associated with the state matrix $\boldsymbol{A}$. Then, the state matrix $\boldsymbol{A}$ can be expressed as

$$\boldsymbol{A}=\boldsymbol{\Psi}\boldsymbol{Z}\boldsymbol{\Psi}^{-1}\ . \tag{5}$$

Substituting Eq. (5) into Eq. (3), the following equation can be derived:

$$\boldsymbol{T}_\tau = \boldsymbol{C}_m \boldsymbol{Z}^{\tau-1} \boldsymbol{G}_m, \tag{6}$$

where $\boldsymbol{C}_m = \boldsymbol{C\Psi}$ and $\boldsymbol{G}_m = \boldsymbol{\Psi}^{-1}\boldsymbol{G}$. Note that $\boldsymbol{C}_m$ contains the mode shapes of the system.

MERA/DC is presented for modal-parameter identification from stationary ambient vibration data. By setting up a correlation-function matrix of ambient responses of structure subjected to stationary white noise and introducing an appropriate matrix factorization, modal parameters of a system can be identified through SVD and eigenvalue analysis.

4. Numerical simulation and discussion

To demonstrate the feasibility and effectiveness of the proposed method, a linear 6-dof chain model with viscous damping was considered. A schematic representation of this model is shown in Fig. 1. The mass matrix $\boldsymbol{M}$, stiffness matrix $\boldsymbol{K}$, and the damping matrix $\boldsymbol{D}$ of the system are given as follows:

$$\mathrm{M} = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 \end{bmatrix} \mathrm{N \cdot s^2/m},$$

$$\boldsymbol{K} = 600 \cdot \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & -1 & 3 & -2 \\ 0 & 0 & 0 & 0 & -2 & 5 \end{bmatrix} N/m,$$

$$\boldsymbol{D} = 0.05\boldsymbol{M} + 0.001\boldsymbol{K} + 0.2 \begin{bmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 1 \end{bmatrix}_{6\times 6} N \cdot s/m.$$

Since the damping matrix $\boldsymbol{D}$ cannot be expressed as a linear combination of $\boldsymbol{M}$ and $\boldsymbol{K}$, the system has non-proportional damping (complex modes in general). Consider that the ambient vibration input is a practical vibration recorded at Fong-Yung Elementary School in Taidong on September 21, 1999, when the Chi-Chi Earthquake with a moment magnitude of 7.6 occurred in central Taiwan. The sampling interval and period of this seismic record were $\Delta t = 0.004\,s$ and $T = 427.996$ s, respectively. A sample of the seismic record, which serves as the excitation input acting on the sixth mass point of the model, is shown in Fig. 2. The displacement responses of the system were obtained using Newmark's method. Then, modal-parameter identification was performed using the simulation responses.

For data with sufficiently long duration, the correlation - function matrix $\boldsymbol{T}_\tau$ of the system responses can be estimated from a single sample function of time (ergodicity assumption). When the correlation-function matrix $\boldsymbol{T}_\tau$ was evaluated using Eq. (2), the data-correlation matrix $\overline{\boldsymbol{R}}(k)$ and a generalized Hankel matrix $\overline{\boldsymbol{U}}(k)$ can be constructed further. Thus, the state matrix $\boldsymbol{A}$ can be estimated through SVD analysis and modal parameters of a system can be identified by solving the eigenvalue problem associated with the state matrix $\boldsymbol{A}$.

In theory, a continuum structure has an infinite number of degrees of freedom and an infinite number of modes. In practice, the number of modes required to describe the dynamical behavior of the observed structural system is not known. However, the number of important modes of the system under consideration can be roughly determined by using SVD analysis. The number of non-zero singular values is the rank of the generalized Hankel matrix $\overline{\boldsymbol{U}}(k)$; it also corresponds to the

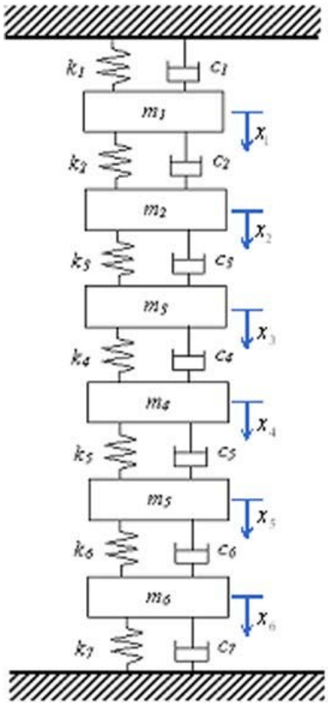

Fig. 1. Schematic plot of the 6-dof chain system.

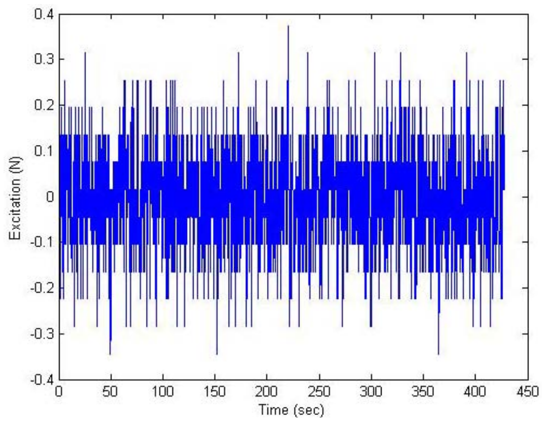

Fig. 2. A recorded sample of the Chi-Chi Earthquake.

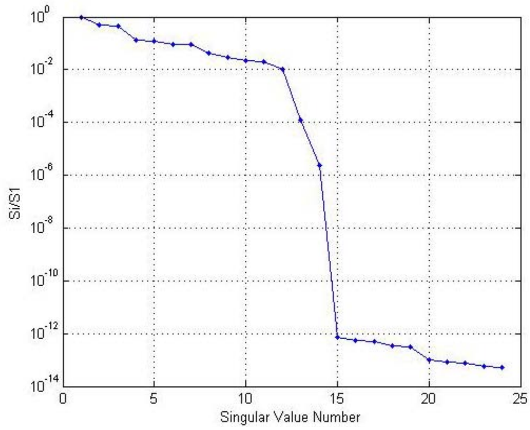

Fig. 3. Singular values associated with the responses corresponding to a recorded sample of the Chi-Chi Earthquake. The distribution of the singular values shows an obvious drop around the singular value number 12, which determines system order and the number of modes to be identified.

order of the system. By examining the distribution of the singular values associated with the system responses, system order and the number of modes to be identified can be determined. The result is shown in Fig. 3, in which the order of the system model was determined to be 12.

The modal frequencies and damping ratios can be obtained by eigen-analysis of the state matrix $\boldsymbol{A}$, and the mode shapes can be obtained from $\boldsymbol{C}_m$ as given in Eq. (6). The results of modal-parameter identification are summarized in Table 1, which shows that in general, the errors in natural frequencies are less than 2 % and the error in damping ratios are less than 15%. The "exact" modal damping ratios listed in Table 1, as well as the exact mode shapes, are actually the equivalent values obtained by using ERA/DC from the simulated free-vibration data of the nonproportionally damped structure.

Furthermore, the modal assurance criterion (MAC) [9], which has been extensively to confirm the vibrating modes of a structural system in experimental modal analysis, was used to check for agreement between the identified mode shapes and exact shapes. The definition of MAC is

$$\mathrm{MAC}(\boldsymbol{\phi}_{iA}, \boldsymbol{\phi}_{jX}) = \frac{\left|\boldsymbol{\phi}_{iA}{}^{T} \boldsymbol{\phi}_{jX}{}^{*}\right|^2}{\boldsymbol{\phi}_{iA}{}^{T} \boldsymbol{\phi}_{iA}{}^{*} \boldsymbol{\phi}_{jX}{}^{T} \boldsymbol{\phi}_{jX}{}^{*}}, \quad (7)$$

where $\boldsymbol{\phi}_{iA}$ and $\boldsymbol{\phi}_{jX}$ denote the i-th theoretical and the j-th identified mode shape, respectively. The superscript * denotes the complex conjugate. The value of MAC varies between 0 and 1. When the MAC value is equal to 1, the two vectors $\boldsymbol{\phi}_{iA}$ and $\boldsymbol{\phi}_{jX}$ represent exactly the same mode shape. However, when the two mode shapes are orthogonal with each other, the MAC value is zero. Thus, verification of the vibrating modes of a structural system can be performed by computing the MAC values.

The identified mode shapes are also compared with the exact mode shapes in Fig. 4. If the system has non-proportional damping, then in general, it has complex modes. However, in this case, the imaginary parts of the mode shapes (both identified and exact) are quite small relative to the real parts, so in Fig. 4 the mode shapes present only the real parts form of the modal-shape vectors. The errors of identified damping ratios and mode shapes are somewhat higher because in general, system response has a lower sensitivity to these modal parameters than to the modal frequencies. Moreover, in the present method, the correlation-function matrix $\boldsymbol{T}_\tau$ is computed directly, in which each row is treated as the impulse response corresponding to each DOF. Thus, by taking advantage of the computation of $\boldsymbol{T}_\tau$, conversion of the original forced-vibration data into free-vibration data and the selection of reference channel for computing correlation [7] are avoided.

Table 1. Results of modal-parameter identification of the 6-DOF system subjected to a recorded sample of the Chi-Chi Earthquake.

Mode	Natural Frequency (rad/s)			Damping Ratio (%)			
	Exact	MERA/DC	Error(%)	Exact	MERA/DC	Error(%)	MAC
1	5.03	5.04	0.26	5.24	4.65	11.23	0.99
2	13.45	13.41	0.26	1.07	1.22	14.35	0.98
3	19.80	19.71	0.44	1.13	1.14	0.99	0.97
4	26.69	26.50	0.70	1.43	1.50	4.84	0.98
5	31.66	31.40	0.82	1.66	1.77	6.90	0.98
6	33.73	33.38	1.03	1.74	1.89	8.38	0.98

To examine further the effectiveness of the present method to a more complex structural system, modal-parameter identification analysis was conducted using a 20-DOF chain model with non-proportional damping subjected to the same practical vibration as in the previous example. Each mass is assumed as 1 kg while all spring constants are 600 N/m. The damping matrix of the system is also assumed to be

$$\boldsymbol{D} = 0.05\boldsymbol{M} + 0.001\boldsymbol{K} + 0.02\begin{bmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 1 \end{bmatrix}_{20\times 20} N\cdot \mathrm{s}/m\,.$$

The considered system has many groups of closely spaced modes, as listed in Table 2. The results of identification are also summarized in Table 2, which shows that the errors in natural frequencies are less than 5 % and the errors in damping are somewhat larger. Observing the MAC values, which signify the consistency between the identified and theoretical mode shapes, 13 out of the 20 modes are identified accurately (MAC ≥ 0.9). The errors of identified damping ratios and mode shapes are somewhat larger because system response generally has a lower sensitivity to these modal parameters than to the modal frequencies. The higher modes are not also identified accurately compared to the lower modes because their contribution to the system response is less than that of the lower modes. Moreover, the proposed method can identify the modal parameters of a structural system subjected to realistic excitation. This may be because under appropriate conditions,

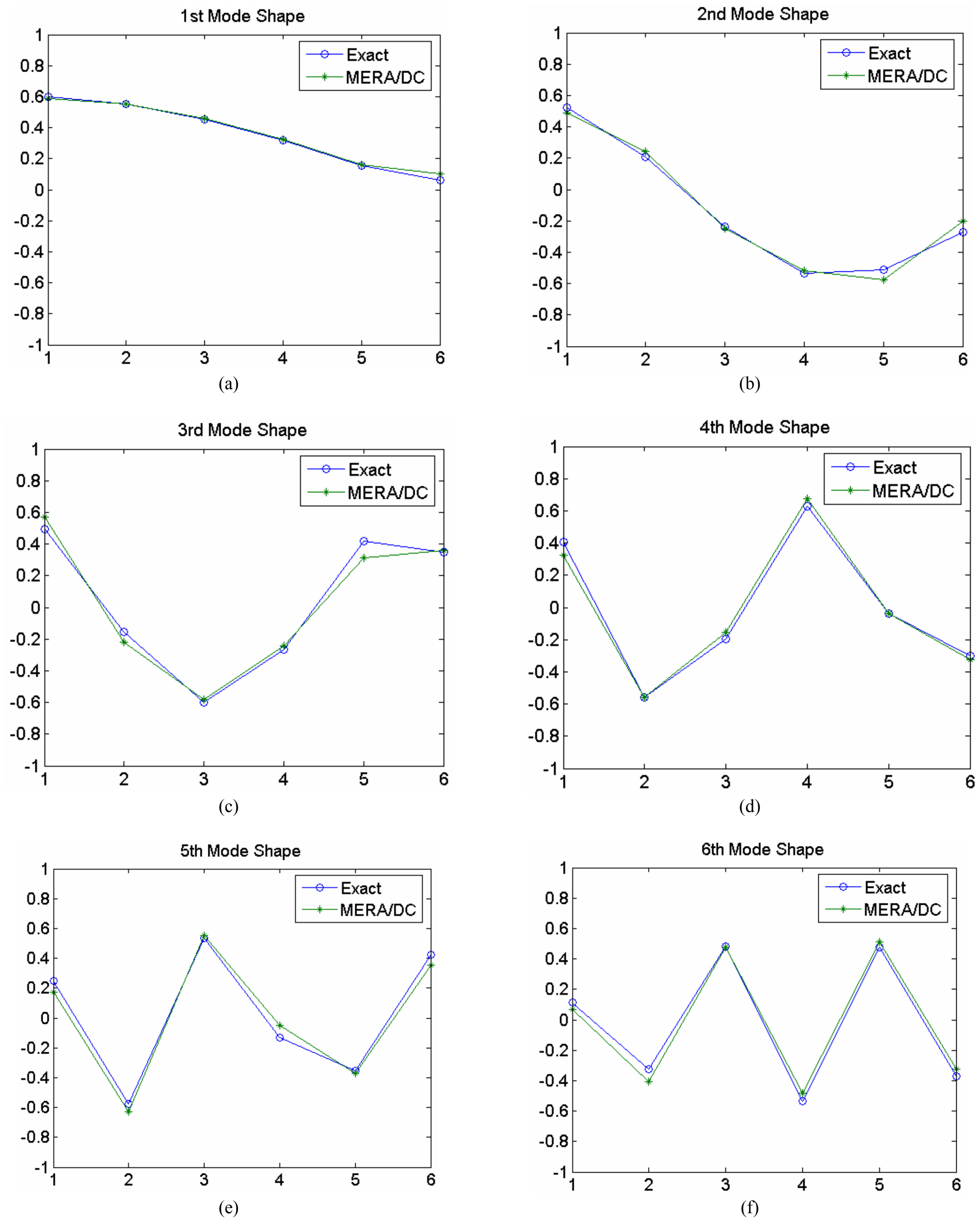

Fig. 4. Comparison between the identified mode shapes and the exact mode shapes of the 6-dof system subjected to a recorded sample of the Chi-Chi Earthquake.

the practical ambient excitation, such as earthquakes, can be approximately modeled as stationary white noise.

5. Conclusions

In this paper, MERA/DC is presented for modal-parameter identification from stationary white-noise ambient vibration data. By setting up a correlation function matrix $\boldsymbol{T}_\tau$ of stationary ambient responses and introducing an appropriate factorization of the correlation-function matrix, modal parameters of the system can be identified by SVD and eigenvalue analysis. Using the MERA/DC proposed the computation of correlation-function matrix $\boldsymbol{T}_\tau$ can be used to avoid the conversion of forced responses into free-vibration data and the selection of reference channel for computing correlation. Through numerical examples of using practical excitation data,

Table 2. Results of modal-parameter identification of a 20-DOF system subjected to a recorded sample of the Chi-Chi Earthquake.

Mode	Natural Frequency (rad/s)			Damping Ratio (%)			
	Exact	MERA/DC	Error(%)	Exact	MERA/DC	Error(%)	MAC
1	3.66	3.68	0.56	5.49	5.65	2.91	0.99
2	7.30	7.30	0.01	0.71	0.70	1.00	0.99
3	10.90	10.89	0.09	0.94	0.94	0.20	0.96
4	14.44	14.42	0.17	0.89	0.89	0.20	0.98
5	17.90	17.85	0.28	1.06	1.06	0.30	0.99
6	21.26	21.18	0.39	1.17	1.17	0.10	0.99
7	24.49	24.37	0.48	1.33	1.33	0.40	1.00
8	27.60	27.42	0.64	1.45	1.45	0.10	1.00
9	30.54	30.31	0.75	1.59	1.59	0.10	1.00
10	33.32	33.02	0.91	1.71	1.71	0.10	1.00
11	35.91	35.53	1.05	1.83	1.83	0.10	0.93
12	38.30	37.84	1.19	1.94	1.93	0.40	0.91
13	40.48	39.94	1.34	2.03	2.03	0.00	0.93
14	42.43	41.81	1.47	2.12	2.12	0.20	0.86
15	44.14	43.44	1.59	2.20	2.18	0.80	0.74
16	45.60	44.84	1.67	2.26	2.23	1.40	0.77
17	46.81	45.34	3.14	2.26	2.24	1.10	0.77
18	47.76	46.02	3.64	2.20	2.56	16.20	0.43
19	48.44	46.86	3.26	2.51	2.49	0.80	0.40
20	48.85	47.70	2.35	2.41	1.00	58.51	0.35

the present method has been shown to be valid and robust in the identification of modal parameters from stationary ambient vibration data under noisy condition.

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Nomenclature

$\boldsymbol{A}$: State matrix with dynamic characteristics of the system
$\boldsymbol{C}$: Output influence matrix for the state vector
$\boldsymbol{D}$: Damping matrix
$\boldsymbol{K}$: Stiffness matrix
$\boldsymbol{M}$: Mass matrix
$\overline{\boldsymbol{R}}(k)$: Data -correlation matrix
$R_{ij}(\tau)$: Cross-correlation function between two stationary response signals $x_i(t)$ and $x_j(t)$
p : Number of the channel of the measured response
$\boldsymbol{T}_\tau$: Correlation- function matrix
$\boldsymbol{Y}_k$: Measured system responses at the time -step k
$\overline{\boldsymbol{U}}(k)$: Generalized Hankel matrix
$\boldsymbol{u}_k$: Zero-mean stationary white noise at time -step k
ϕ_{ir} : i-th component of the r-th mode shape

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